UNIVERSITI MALAYA
UNIVERSITY OF MALAYA

PEPERIKSAAN IJAZAH SARJANA MUDA KEJURUTERAAN
EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING

SESI AKADEMIK 2018/2019 : SEMESTER II
ACADEMIC SESSION 2018/2019 : SEMESTER II

KIX1002 : Matematik Kejuruteraan 2
Engineering Mathematics 2

Jun 2019 — Masa : 2 jam
June 2019 — Time : 2 hours

ARAHAN KEPADA CALON:
INSTRUCTIONS TO CANDIDATES:

Jawab **KESEMUA** Soalan
*Answer **ALL** Questions*

(Kertas soalan ini mengandungi 4 soalan dan 1 lampiran dalam 9 halaman yang dicetak)
(This question paper consists of 4 questions and 1 Appendix on 9 printed pages)

Soalan 1
Question 1

(a) Pertimbangkan satu medan elektrik yang berasal dari satu peranti dan merambat melalui ruang bebas di dalam tiga dimensi. Vektor medan elektrik, $\overline{E}$ pada bila-bila masa diberikan oleh

Consider an electric field originating from a device and travelling through free space in three dimensions. The electric field vector, $\bar{E}$ at any instant of time is given by:

$$\bar{E}\,(x,y,z) = (x^3 + y^2 + z)\hat{\imath} + (ze^x)\hat{\jmath} + (xyz - 9xz)\hat{k}$$

i. Tentukan putaran vektor medan elektrik, $\overline{E}$ berdekatan dengan titik (0, 1, 3)

Determine the rotation of electric field vector, $\bar{E}$ near the point (0, 1, 3).

(6 markah / *marks*)

ii. Tukar vektor medan elektrik, $\overline{E}$ pada bila-bila masa ke dalam medan skalar menggunakan kecapahan.

Convert electric field vector, $\bar{E}$ at any instant of time into scalar field using divergence.

(4 markah / *marks*)

iii. Perlu diingat bahawa vektor medan elektrik dan medan magnet adalah **tegak lurus** antara satu sama lain untuk gelombang satah di dalam ruang bebas. Oleh kerana itu, sila **sahkan dan bincangkan** bahawa vektor medan elektrik, $\overline{E}$ dan vektor medan magnet, $\overline{H}$ bagi $(2\cos(xz) - 2y^2)\hat{\imath} + (4z^2 \;+ 2e^x)\hat{\jmath} + (\;e^{2x}\sin(yz)\hat{k}$ adalah tegak lurus di antara satu sama lain pada titik (0, 2, 2).

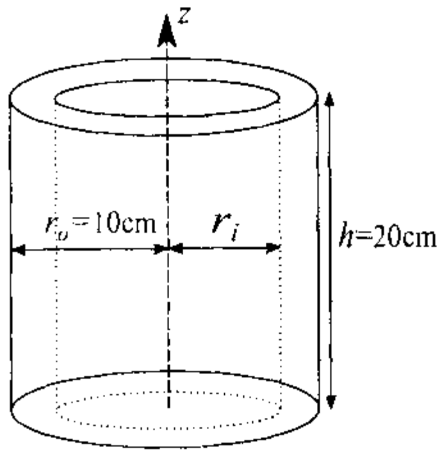
*It should be noted that electric field and magnetic field vectors are **perpendicular** to each other for a plane wave in free space. Therefore, please **verify and discuss** that electric field vector, $\bar{E}$ and the magnetic field vector, $\bar{H}$ of $(2cos(xz) - 2y^2)\hat{\imath} + (4z^2 \;+ 2e^x)\hat{\jmath} + (\;e^{2x}sin(yz)\hat{k}$ are perpendicular to each other at point (0,2,2).*

(5 markah / *marks*)

Soalan 2
Question 2

(a) Momen inersia untuk satu objek berketumpatan malar mengikut paksi z boleh diungkapkan sebagai $I_z = \int_V \rho r^2 \, dV$, di mana ρ ialah ketumpatan dan r ialah jarak radial dari paksi z. Satu silinder berongga dalam Rajah S2(a) mempunyai radius luaran $r_o = 10\text{cm}$, tinggi $h = 20\text{cm}$ dan ketumpatan $\rho = 1\text{g/cm}^3$. Apakah radius dalaman, r_i, jika anda perlu merekabentuk silinder tersebut supaya mempunyai momen inersia $I_z = 75990\pi \text{ gcm}^2$? Nota: Guna pengamiran isipadu dalam sistem koordinat polar silinder untuk mendapatkan jawapan.

Moment of inertia of a constant density object with respect of z axis can be express as $I_z = \int_V \rho r^2 \, dV$, where ρ is the density and r is the radial distance from z axis. A hollow cylinder in Figure Q2(a) has the outer radius of $r_o = 10cm$, the height of $h = 20cm$ and the density of $\rho = 1g/cm^3$. What is the inner radius, r_i, if you are required to design the cylinder so that the moment of inertia is $I_z = 75990\pi \ gcm^2$? Note: Use volume integral in cylindrical polar coordinate system to derive your answer.



Rajah S2(a) / *Figure Q2(a)*

(7 markah/ *marks*)

(b) Buktikan teorem Stoke untuk medan vector $F(x, y, z) = \langle xz, xy, \frac{y^2}{2} \rangle$ dan S adalah bahagian paraboloid $z = 1 - x^2 - y^2$ di atas satah xy (persamaan satah $z = 0$).

Verify Stoke's theorem for the vector field $F(x, y, z) = \langle xz, xy, \frac{y^2}{2} \rangle$ and S is the part of the paraboloid $z = 1 - x^2 - y^2$ above the xy-plane (plane equation $z = 0$).

(8 markah/*marks*)

SOALAN 3
QUESTION 3

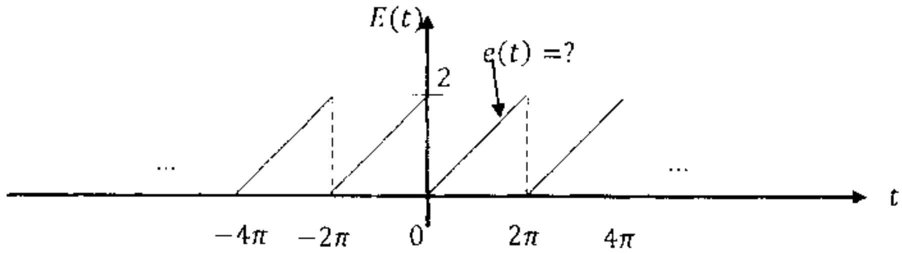
Satu litar elektrik RLC digerakkan oleh satu voltan berkala, $E(t)$. Persamaan menakluk bagi system ini didasarkan oleh:
A RLC electrical circuit is powered by a periodic voltage, $E(t)$. The governing equation of the system is given by:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$$

yang mana L, R dan C adalah masing-masing induktan, rintangan dan kapasitan. q adalah caj elektrik.
where L, R and C are the inductance, resistance and capacitance respectively. q is the electrical charge.

(a) Tentukan fungsi $e(t)$ dalam lingkungan $0 < e(t) < 2\pi$ dan frekuensi sudut asas, ω bagi voltan berkala , $E(t)$ dengan berpandukan Rajah S3(a). Justeru, tentukan perwakilan siri Fourier bagi $E(t)$. [Panduan: Pelajar boleh menggunakan sifat ganjil, genap dan kelinearan untuk mendapatkan pekali Fourier dengan pengiraan yang lebih mudah]
Determine the function $e(t)$ within $0 < e(t) < 2\pi$ and the fundamental angular frequency, ω of the periodic voltage, $E(t)$ by referring to Figure Q3(a). Hence, determine the Fourier series representation of $E(t)$. [Hint: Student can apply odd, even or linearity property to obtain the Fourier coefficient with simpler calculation]

(8 markah/*marks*)



Rajah S3(a) / *Figure Q3(a)*

(b) Dengan menganggap penyelesaian khas,
By assuming the particular solution,

$$q_p = A_0 + \sum_{n=1}^{\infty}(A_n \cos nt) + \sum_{n=1}^{\infty}(B_n \sin nt)$$

Selesaikan ODE di bawah dan dapatkan pekali A_0, A_n dan B_n bagi penyelesaian khas itu.
Solve the ODE below and obtain the coefficients A_0, A_n and B_n of the particular solution.

$$\frac{d^2q}{dt^2} + 40\frac{dq}{dt} + 15q = 1 - \sum_{n=1}^{\infty}\left(\frac{2}{n\pi}\sin nt\right)$$

(7 markah/*marks*)

Soalan 4
Question 4

(a) Persamaan Laplace bagi satu plat segiempat tepat diberi sebagai

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 1,\ 0 < y < 1$$

dan syarat-syarat sempadan adalah

$$u(0, y) = 0, \qquad u(1, y) = 0$$
$$u(x, 0) = 100, \qquad u(x, 1) = 200$$

Dengan menggunakan $u = XY$ dan $-\lambda$ sebagai pemalar pembahagi, dapat diperoleh dua persamaan perbezaan biasa lelurus $X'' + \lambda X = 0$ and $Y'' - \lambda Y = 0$.

The Laplace's equation for a rectangular plate is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 1,\ 0 < y < 1$$

and the boundary conditions are

$$u(0, y) = 0, \qquad u(1, y) = 0$$
$$u(x, 0) = 100, \qquad u(x, 1) = 200$$

Using $u = XY$ and $-\lambda$ as a separation constant, we obtain two linear ordinary differential equations $X'' + \lambda X = 0$ and $Y'' - \lambda Y = 0$.

i. Dengan mempertimbangkan semua kes berkemungkinan bagi λ, selesaikan persamaan-persamaan perbezaan untuk mendapatkan nilai eigen dan fungsi eigen.

   By considering all possible cases for λ, solve the differential equations to obtain the eigenvalue and eigenfunction.

   (5 markah / *marks*)

ii. Selesaikan masalah nilai sempadan dengan menggunakan siri Fourier. Lengkapkan pengamiran untuk mencari pekali-pekali Fourier.

   Solve the boundary value problem using Fourier series. Complete the integration to find the Fourier coefficients.

   Diberi / *Given:* $\quad cos(n\pi) = (-1)^n$

   (6 markah / *marks*)

(b) Pertimbangkan persamaan gelombang satu dimensi

Consider the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

i. Tunjukkan bahawa

$$u(x, t) = u_0\, sin(2x)\, cos(2ct)$$

merupakan penyelesaian kepada persamaan gelombang yang mewakili satu tali bergetar dengan panjang $L = \frac{\pi}{2}$.

Show that

$$u(x,t) = u_0 \sin(2x) \cos(2ct)$$

is the solution to the wave equation which represents a vibrating string of length $L = \frac{\pi}{2}$.

(2 markah / *marks*)

ii. Pemalar c dalam persamaan gelombang diberi sebagai

$$c = \sqrt{T/\rho}$$

di mana ρ ialah jisim seunit panjang and T ialah magnitud tegangan dalam tali. Apabila anda mengetatkan tali suatu alat muzik seperti gitar, piano atau selo, not (daripada mod u) menjadi tinggi. Apa yang berubah dalam persamaan perbezaaan?

The constant c in the wave equation is given by

$$c = \sqrt{T/\rho}$$

where ρ is mass per unit length and T is the magnitude of the tension in the string. When you tighten the string of a musical instrument such as a guitar, piano, or cello, the note (from the mode u) gets higher. What has changed in the differential equation?

(2 markah / *marks*)

Lampiran A / *Appendix A*

Perwakilan parametrik bagi silinder:
Parametric representation of cylinder:

$$x^2 + y^2 = a^2$$
$$r(u,v) = a\cos u\ \boldsymbol{i} + a\sin u\ \boldsymbol{j} + v\ \boldsymbol{k}$$

Perwakilan parametrik bagi sfera:
Parametric representation of sphere:

$$x^2 + y^2 + z^2 = a^2$$
$$r(u,v) = a\cos v\cos u\ \boldsymbol{i} + a\cos v\sin u\ \boldsymbol{j} + a\sin v\ \boldsymbol{k}$$

Perwakilan parametrik bagi kon:
Parametric representation of cone:

$$z = \sqrt{x^2 + y^2}$$
$$r(u,v) = u\cos v\ \boldsymbol{i} + u\sin v\ \boldsymbol{j} + u\ \boldsymbol{k}$$

Koordinat Cartesian ke Sfera
Cartesian to Spherical coordinates

$$x = r\sin\theta\cos\emptyset$$
$$y = r\sin\theta\sin\emptyset$$
$$z = r\cos\theta$$
$$dS = r^2\sin\theta\ d\theta d\emptyset$$
$$dV = r^2\sin\theta\ d\theta d\emptyset dz$$

Koordinat Cartesian ke Silinder
Cartesian to Cylindrical coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$
$$dV = rdrd\theta dz$$

Trigonometric Identities

$$\sin x\sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$
$$\cos x\cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$$
$$\sin x\cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos^2 x = \frac{1}{2}[1 + \cos 2x]$$
$$\sin^2 x = \frac{1}{2}[1 - \cos 2x]$$

Jadual Jelmaan Laplace
Laplace Transform Table

$f(t)$	$F(s)$
1	$\frac{1}{s}$
$\delta(t)$	1
t	$\frac{1}{s^2}$
$t^n,\ n = 1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
te^{at}	$\frac{1}{(s-a)^2}$
$t^n e^{at},\ n = 1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$

$f(t)$	$F(s)$
$af(t) + bg(t)$	$aF(s) + bG(s)$
$u(t-a)$	$\frac{e^{-as}}{s}$
$\delta(t-a)$	e^{-as}
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$e^{at}f(t)$	$F(s-a)$
$\frac{df}{dt}$	$sF(s) - f(0)$
$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}F(s)$
$tf(t)$	$-\frac{d}{ds}F(s)$
$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
$f(t) * g(t)$	$F(s)G(s)$

The Fourier series of a function *f* defined on the interval (-*p*, *p*) is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{p}x + b_n \sin\frac{n\pi}{p}x\right)$$

where

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(x)dx$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi}{p}x\,dx$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi}{p}x\,dx$$

The Fourier Cosine series of an even function on the interval (-*p*, *p*) is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p}x$$

where

$$a_0 = \frac{2}{p}\int_0^p f(x)dx$$

$$a_n = \frac{2}{p}\int_0^p f(x)\cos\frac{n\pi}{p}x\,dx$$

The Fourier Sine series of an odd function on the interval (-*p*, *p*) is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{p}x$$

where

$$b_n = \frac{2}{p}\int_0^p f(x)\sin\frac{n\pi}{p}x\,dx$$

Fourier series, $\mathbf{F(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega x + b_n \sin n\omega x)}$ **where**

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos n\omega x\;dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin n\omega x\,dx$$

TAMAT

END