UNIVERSITI MALAYA
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PEPERIKSAAN IJAZAH SARJANA MUDA KEJURUTERAAN
EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING

SESI AKADEMIK 2022/2023 : SEMESTER II
ACADEMIC SESSION 2022/2023 : SEMESTER II

KIX1002 : Matematik Kejuruteraan 2
Engineering Mathematics 2

Julai 2023 Masa : 2 jam
July 2023 Time : 2 hours

ARAHAN KEPADA CALON:
INSTRUCTIONS TO CANDIDATES:

Calon dikehendaki menjawab semua soalan.
Candidate must answer all the questions.

(Kertas soalan ini mengandungi 3 soalan dalam 9 halaman yang dicetak)
(This question paper consists of 3 questions on 9 printed pages)

Soalan 1
Question 1

(a) Kaedah siri kuasa adalah keadah asas biasa untuk menyelesaikan persamaan bezaan linear dengan pekali berubah. Tentukan jejari dan jeda penumpuan siri kuasa tersebut.

The power series method is the standard basic method for solving linear differential equations with variable coefficients. Determine the radius and interval of convergence of the following power series.

(i) $\sum_{k=0}^{\infty} \frac{k(x+4)^k}{5^{k+1}}$

(ii) $\sum_{n=0}^{\infty} \frac{n(x-7)^n}{n!}$

(5 markah/*marks*)

(b) Tentukan titik-titik tunggal bagi persamaan bezaan tersebut. Bagi setiap titik tunggal, jelaskan sama ada ia adalah titik tunggal biasa atau tidak biasa.

Determine the singular points of the following differential equations. For each singular point, explain whether it is regular or irregular singular point.

(i) $(x^2 - 16)^2 y'' + (x + 4)y' + 2y = 0$

(ii) $(3x^2 - 11x - 4)y'' + (3x + 1)y' + \frac{6}{x-4} y = 0$

(5 markah/*marks*)

Soalan 2
Question 2

(a) Satu sesondol menggerakkan injap untuk mendapat profil berkala $f(t)$ sebagai fungsi dengan kalaan 2π seperti:

A cam actuates a valve to get periodic profile $f(t)$ as a function of period 2π such that:

$$f(t) = t^2 \;\; ; -\pi < t < \pi$$

(i) Lakarkan graf bagi $f(t)$ dalam jeda $-3\pi < t < 3\pi$.

Sketch a graph of $f(t)$ in the interval $-3\pi < t < 3\pi$.

(1 markah/*marks*)

(ii) Tunjukkan siri Fourier bagi $f(t)$ dalam jeda $-\pi < t < \pi$ adalah seperti yang ditunjukkan. (Nota: a_0, a_n, b_n perlu dikira)

Show that the Fourier series for $f(t)$ in the interval $-\pi < t < \pi$ is as shown. (Note: a_0, a_n, b_n have to be calculated)

$$\frac{\pi^3}{3} - 4[\cos t - \frac{1}{2^2}\cos 2t + \frac{1}{3^2}\cos 3t - \frac{1}{4^2}\cos 4t + \cdots]$$

(14 markah/*marks*)

(iii) Dengan memberikan nilai *t* yang sesuai, tunjukkan

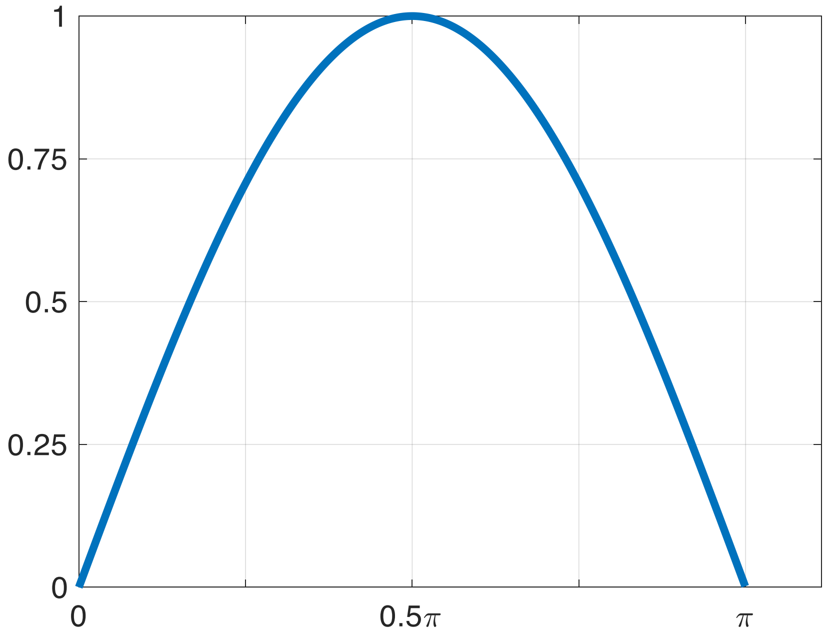
By giving an appropriate value to t, show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}\cdots$$

(2 markah/*marks*)

(b) Ditunjukkan dalam Rajah S2b adalah fungsi tidak berkala $h(x) = \sin x$ dalam julat $0 < x < \pi$ yang boleh digunakan untuk mewakili bonggol jalan.

Shown in Figure Q2b is non-periodic function $h(x) = \sin x$ in the range of $0 < x < \pi$ which can be used to represent a road bump.



Rajah S2b/*Figure Q2b*

(i) Lakarkan fungsi terhasil apabila Pengembangan Separa Julat Siri Fourier Kosinus dibuat ke atas fungsi $h(x)$.

Sketch the resulting function when Half Range Fourier Cosine Series Expansion is performed onto $h(x)$.

(1 markah/*marks*)

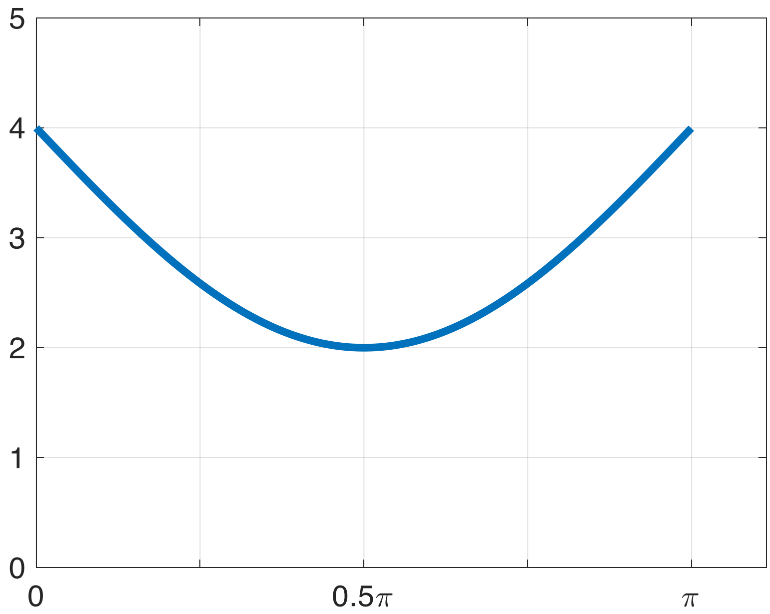
(ii) Berdasarkan plot yang dilakar dalam bahagian b(i) di atas, kenal pastikan sama ada fungsi $h(x)$ dalam kalaan 2π adalah fungsi ganjil atau genap dan berikan alasan anda secara analitik (matematik). Kemudiannya, terangkan mengapa satu (atau lebih) daripada pekali Fourier adalah kosong.

Based on the sketched plot in part b(i) above, identify whether the expanded $h(x)$ function within a period of 2π is odd or even function and justify your answer analytically (mathematically). Then, explain why one (or more) of the Fourier coefficient(s) is (are) zero.

(3 markah/*marks*)

(iii) Ditunjukkan dalam Rajah S2b(iii) adalah satu fungsi sinusoidal tidak berkala $g(x)$ dalam julat $0 < x < \pi$. Tentukan pertalian matematik di antara fungsi $g(x)$ dan $h(x)$. Sekiranya Pengembangan Separa Julat Siri Fourier Kosinus bagi $h(x)$ diwakili oleh Persamaan S2b, simpulkan Pengembangan Separa Julat Siri Fourier Kosinus bagi $g(x)$.

Shown in Figure Q2b(iii) is a non-periodic sinusoidal function of $g(x)$ in the range of $0 < x < \pi$. Determine the mathematical relationship between the function $g(x)$ and $h(x)$. If the Half Range Fourier Cosine Series Expansion for $h(x)$ is represented by Equation Q2b, deduce the Half Range Fourier Cosine Series Expansion for $g(x)$.



Rajah S2b(iii)/*Figure Q2b(iii)*

$$h(x) = \frac{2}{\pi} + \frac{\sum_{n=1}^{\infty} \cos(nx)\,(-(-1)^n - (-1)^n - 2)}{\pi(n+1)(n-1)}$$

Persamaan S2b/*Equation Q2b*

(4 markah/*marks*)

Soalan 3
Question 3

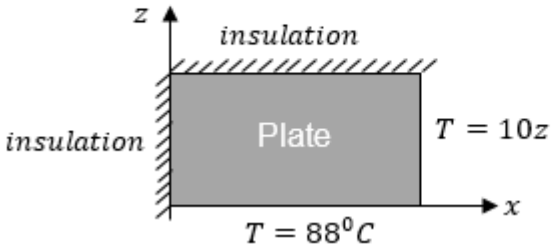
(a) Persamaan Laplace bagi satu plat segiempat diberi bawah, dimana $T(x,z) =$ taburan suhu untuk plat tersebut (Rajah 3).

The Laplace's equation for a rectangular plate is given below, where $T(x,z) =$ temperature distribution of the plate (Figure 3).

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = 0, \qquad 0 < x < 10,\ \ 0 < z < 5$$

(i) Rumuskan syarat sempadan untuk plat segiempat.

Formulate the boundary condition (BC) of the rectangular plate.



Rajah S3/*Figure Q3*

(4 markah/*marks*)

(ii) Carikan penyelesaian am untuk persamaan Laplace bagi setiap kes.

Find the general solution of the Laplace's equation for each case.

(9 markah/*marks*)

(b) Sambungkan bahagian (a)(ii) dan selesaikan selanjutnya jumlah penyelesaian am untuk persamaan Laplace, tetapi tertakluk kepada syarat sempadan berikut.

Continue part (a)(ii) and solve further the total general solution of the Laplace's equation, but subjected to the following boundary conditions.

BC 1 & 2: $\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0\ ,\ \ \left.\frac{\partial T}{\partial z}\right|_{z=5} = 0 \qquad 0 < x < 10\ ,\ 0 < z < 5$

(7 markah/*marks*)

(c) Andaikan persamaan Laplace di bahagian (b) mempunyai jumlah penyelesaian am berikut. Terus kirakan penyelesaian khusus yang tertakluk kepada syarat sempadan berikut. Dengan itu, anggarkan suhu, $T(x, z) = T(5{,}2)$.

Assume the Laplace's equation in part (b) has the following total general solution. Continue to determine the particular solution, which is subjected to the following boundary conditions. Then, estimate the temperature, $T(x, z) = T(5{,}2)$.

$$T(x,z) = (A_1 + B_1 x) + \sum_{n=1}^{\infty} \left(cos\,(\frac{n\pi}{5} z) \right) \left(A_{2,n} cosh\,(\frac{n\pi}{5} x) + B_{2,n} sinh\,(\frac{n\pi}{5} x) \right)$$

BC 3 & 4: $T(0, z) = 100$, $T(10, z) = 0$, $\quad 0 < x < 10$, $0 < z < 5$

(5 markah/*marks*)

Lampiran A
Appendix A

D'Alembert's Ratio for Power Series

$$\rho = \lim_{n\to\infty} \frac{u_{n+1}}{u_n}$$

Standard Form of Differential Equation with Variable Coefficients

$$y'' + P(x)y' + Q(x)y = 0$$

Lampiran B
Appendix B

Fourier Series and Half-Range Fourier Series Expansion

(i) The Fourier Series of a function $f(x)$ defined on the interval $(-\infty,\infty)$ is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega x + b_n \sin n\omega x)$$

where

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\, dx$$
$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\, cos\, n\omega x\ \ dx$$
$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\, sin\, n\omega x\, dx$$

(ii) The Half-Range Fourier Cosine Series Expansion of a function $f(x)$ defined on the interval $(0,\tau)$ is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega x)$$

where

$$a_0 = \frac{1}{L}\int_{0}^{\tau} f(x)\, dx$$
$$a_n = \frac{2}{L}\int_{0}^{\tau} f(x)\, cos\, n\omega x\ \ dx$$

(iii) The Half-Range Fourier Sine Series Expansion of a function $f(x)$ defined on the interval $(0,\tau)$ is given by

$$f(x) = \sum_{n=1}^{\infty} (b_n \sin n\omega x)$$

where

$$b_n = \frac{2}{L} \int_0^{\tau} f(x) \, sin \, n\omega x \;\; dx$$

TAMAT
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