

# KIX1002: ENGINEERING MATHEMATICS 2

## TUTORIAL 10: FOURIER SERIES

1. Find the fundamental period of the following function:

a)  $\cos 2x$

b)  $\sin 2\pi x$

c)  $\cos nx$

d)  $\sin \frac{2\pi nx}{k}$

2. Sketch the given function and find the Fourier Series.

a)  $f(x) = \begin{cases} -1 & ; -\pi < x < 0 \\ 2 & ; 0 < x < \pi \end{cases} ; p = 2\pi$

b)  $f(x) = \begin{cases} 0 & -2 < x < -1 \\ -2 & -1 \leq x < 0 \\ 1 & 0 \leq x < 1 \\ 0 & 1 \leq x < 2 \end{cases} ; p = 4$

c)  $f(x) = \begin{cases} 1 & ; -1 < x < 0 \\ x & ; 0 < x < 1 \end{cases} ; p = 2$

3. Even and Odd Functions. Sketch the given functions,  $f(x)$ . Determine whether it is an even, odd or neither odd nor even. For part (a) and (b), find the appropriate Fourier Cosine or Fourier Sine Series.

a)  $(x) = |x| ; -\pi < x < \pi ; f(x) = f(x + 2\pi n), n = \text{integer}$

b)  $f(x) = x ; -1 < x < 1 ; f(x) = f(x + 2n), n = \text{integer}$

c)  $f(x) = x^2 ; -1 < x < 1 ; f(x) = f(x + 2n), n = \text{integer}$

(Do not find the Fourier Series for 3c)

d)  $f(x) = e^x ; -\pi < x < \pi ; f(x) = f(x + 2\pi n), n = \text{integer}$

(Do not find the Fourier Series for 3d)

4. Solve the following questions:

a) Obtain the Fourier series for a periodic function  $f(t)$  with period  $2\pi$  :

$$f(t) = t^2, \quad -\pi < t < \pi$$

b) Obtain the Fourier series for a periodic function  $f(t)$  with period  $2\pi$  :

$$f(t) = t, \quad -\pi < t < \pi$$

c) Differentiate the Fourier series in (a) to obtain  $f'(t)$

d) Find the Fourier series of  $f(t) = t$  by using result in part (c) and compare it with (b).

5. Consider the following ODE which represents an undamped mass-spring system:

$$\frac{1}{16} \frac{d^2x}{dt^2} + 4x = f(t)$$

where  $f(t)$  is a periodic function as shown in Fig. 1. Obtain a particular solution for the ODE.

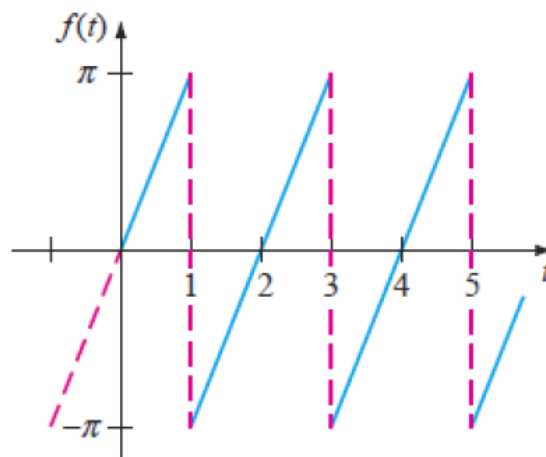


Fig. 1. Periodic function  $f(t)$