KIX 1002: ENGINEERING MATHEMATICS 2

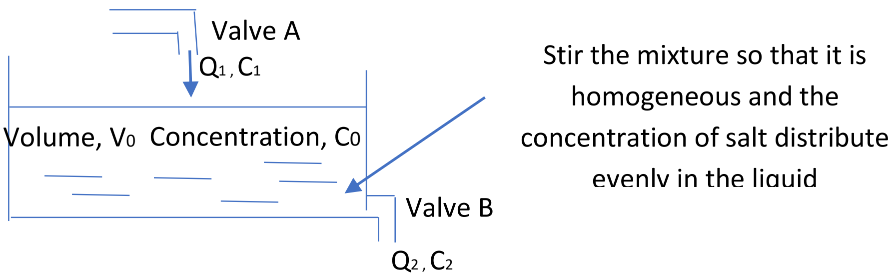
Tutorial 5: Engineering Applications of Differential Equations

1. (a) Given the population of rabbit in human habitat (rabbit farm) grows at a rate proportional to the number of rabbit at time t (year). It is observed that 200 and 800 rabbits are presented at 3rd year and 6th year respectively. What was the initial number of the rabbit, $y(0) = y_0$? How long does it take the population to double to $2y_0$?

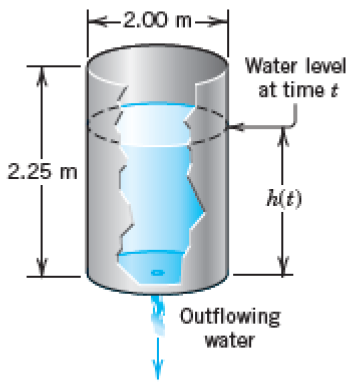
   (b) A person has bought 1000 rabbits from the farm and releases them to the jungle that is full of predator (i.e. $p = 50$ snakes at the time he/she releases the rabbit). Given the governing equation of the rabbit population is changed to $\frac{dy}{dt} = -p + 3y(1 - \frac{y}{100})$. How long does it take the rabbit population to decrease to half?

   Hint: The doubling time decrease/increase to its initial amount/quantity is related to half-life concept. Students are encouraged to study its application in engineering. For example, to compare the charging time performance of various capacitors using half-life indicator in electrical field.

2. A brine mixing problem is illustrated in the following figure where a tank contains a liquid of volume $V_0 = 10m^3$ with concentration $C_0 = 0.5\frac{g}{m^3}$ initially and two valves (A & B) are opened simultaneously. The rate of change for the amount of salt in the tank over time is given: $\frac{dx}{dt} = Q_1C_1 - Q_2\frac{x(t)}{V_0 + (Q_1 - Q_2)t}$. Let $x(t)$ = amount of salt; concentration of salt over time= $C(t) = \frac{x(t)}{V(t)}$; $Q_1 = Q_2 = 2\frac{m^3}{\min}$; $C_1 = 1\frac{g}{m^3}$. What is the change of the amount of salt and also the change of its concentration over time.



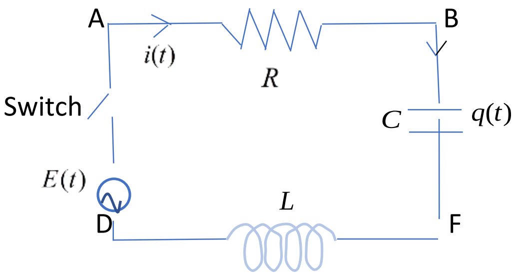
3. The figure below shows the outflow of water from a cylindrical tank with a hole at the bottom. Determine the height of the water in the tank at any time if the tank has a diameter of 2m,, the hole has diameter 1cm, and the initial height of the water when the hole is opened is 2.25m. When will the tank be empty?



Tank

4. Given the governing equation for RLC electrical circuit: $L\dfrac{d^2q(t)}{dt^2}+R\dfrac{dq(t)}{dt}+\dfrac{1}{C}q(t)=E(t)$.

An inductor of $L=$ 2 henrys and a resistor of $R=$ 10 ohms are connected in series with an emf of E volts. Note that in this case the capacitor has been removed. At $t=0$, the switch S is closed, thus no charge and current flow at that moment. Find the charge and current at any time $t>0$ if



a) $E(t)=40$ volts

b) $E(t)=20e^{-3t}$ volts

5. Let the governing equation for a vibrating car structure: $2\frac{d^2x(t)}{dt^2}+7\frac{dx(t)}{dt}+8x(t)=\underbrace{F(t)}_{\substack{Forcing\\function}}$, $x(0)=2$, $\dot{x}(0)=0$. Find the total solution for the 2nd order ODE equation if the forcing function is given as follows:

(a) No excitation, $F=0$ and it is subjected to initial condition. Hence prove that the solutions are linearly independent to each other.

(b) Repeat the same problem in 5(a) with various combinations of damping, i.e. $2\frac{d^2x(t)}{dt^2}+8\frac{dx(t)}{dt}+8x(t)=\underbrace{F(t)}_{\substack{Forcing\\function}}$

(c) Repeat the same problem in 5(a) with various combinations of damping, i.e. $2\frac{d^2x(t)}{dt^2}+9\frac{dx(t)}{dt}+8x(t)=\underbrace{F(t)}_{\substack{Forcing\\function}}$

6. Continue the problem 5. Let the governing equation for a vibrating car structure: $2\frac{d^2x(t)}{dt^2}+7\frac{dx(t)}{dt}+8x(t)=\underbrace{F(t)}_{\substack{Forcing\\function}}$, $x(0)=2$, $\dot{x}(0)=0$. Find the total solution for the 2nd order ODE equation if the forcing function is given as follows:

(a) Engine excitation $F(t)=5\cos 10t$
(b) Engine excitation $F(t)=8\sin 8t$
(c) Engine excitation $F(t)=e^{-10t}$
(d) Engine excitation $F(t)=5e^{-10t}\cos 10t$
(e) Road excitation $F(t)=10$
(f) Road excitation $F(t)=5t^2+7t+9$
(g) Road excitation $F(t)=6te^{t}+3t$