KIX1002: ENGINEERING MATHEMATICS 2

TUTORIAL 8: LAPLACE TRANSFORM

1. Using the definition of Laplace transform, find the transform $\mathcal{L}\{f(t)\}$. Show the details of your integration.

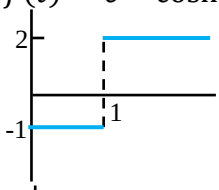
i) $f(t) = 7t - 5$

ii) $f(t) = \sinh kt$

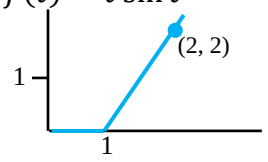
iii) $f(t) = e^{-t}\cosh t$

iv) $f(t) = t\sin t$

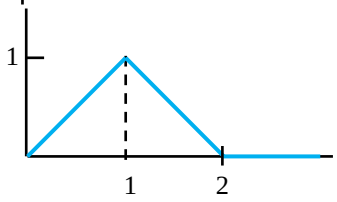
v)



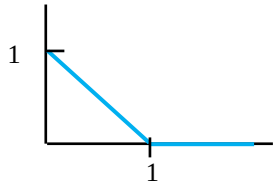
vi)



vii)



viii)



2. Referring to the Laplace transform table, find the transform $\mathcal{L}\{f(t)\}$. Show the modification details that you need to do before the transform can be done.

i) $f(t) = (t+1)^3$

ii) $f(t) = \sin 2t \cos 2t$

iii) $f(t) = ke^{-at}\cos \omega t$

iv) $f(t) = \sin(4t+5)$

v) $f(t) = t^3 e^{-2t}$

vi) $f(t) = (1 - e^t + 3e^{-4t})\cos 5t$

vii) $f(t) = (3t+1)\,u(t-1)$

viii) $f(t) = \sin t\; u\left(t - \frac{\pi}{2}\right)$

3. Given $F(s) = \mathcal{L}\{f(t)\}$, find $f(t)$. Show the details of your work, including the partial fraction expansion if necessary.

i) $\mathcal{L}^{-1}\left(\frac{1}{s^2} - \frac{48}{s^5}\right)$

ii) $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\}$

iii) $\mathcal{L}^{-1}\left(\frac{4s}{4s^2+1}\right)$

iv) $\mathcal{L}^{-1}\left(\frac{2s-6}{s^2+9}\right)$

v) $\mathcal{L}^{-1}\left\{\frac{0.9s}{(s-0.1)(s+0.2)}\right\}$

vi) $\mathcal{L}^{-1}\left\{\frac{s-3}{(s-\sqrt{3})(s+\sqrt{3})}\right\}$

vii) $\mathcal{L}^{-1}\left\{\frac{s}{(s+2)(s^2+4)}\right\}$

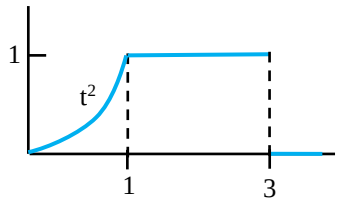
viii) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$

ix) $\mathcal{L}^{-1}\left\{\frac{se^{-\pi s/2}}{s^2+4}\right\}$

x) $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\}$

4.

(a) Using the Laplace transform definition, evaluate the Laplace transform for the following function:



(b) Prove your answer in part (a) by expressing the functions using unit step function and find $\mathcal{L}\{f(t)\}$ by referring to the Laplace transform table.

5. Using the theorem $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{1}{s}F(s)$, find $f(t)$ if $\mathcal{L}\{f(t)\} = \frac{s+1}{s^3+9s}$. You are not allowed to use partial fraction expansion.