KIX1002: ENGINEERING MATHEMATICS 2

TUTORIAL 9: LAPLACE TRANSFORM 2

1. Evaluate the given Laplace transform.

   i) $\mathcal{L}\{t \sinh 3t\}$

   ii) $\mathcal{L}\{te^{-3t} \cos 3t\}$

   iii) $\mathcal{L}\{e^{2t} * \sin t\}$

   iv) $\mathcal{L}\{e^{-t} * e^{t} \cos t\}$

   v) $\mathcal{L}\left\{\int_0^t \tau \sin \tau \, d\tau\right\}$

   vi) $\mathcal{L}\left\{t \int_0^t \sin \tau \, d\tau\right\}$

2. Use the Laplace transform to solve the initial-value problem.

   i) $\frac{dy}{dt} + 3y = 13 \sin 2t\,,\ y(0) = 6$

   ii) $y'' + 16y = f(t),\ y(0) = 0,\ y'(0) = 1$ where

   $$f(t) = \begin{cases} \cos 4t\,, & 0 \le t \le \pi \\ 0, & t \ge \pi \end{cases}$$

   iii) $y' + y = \delta(t-1),\ y(0) = 2$

   iv) $y'' - 7y' + 6y = e^t + \delta(t-2) + \delta(t-4),\ \ y(0) = 0,\ y'(0) = 0$

   v) $y'' + y = \delta\left(t - \frac{1}{2}\pi\right) + \delta\left(t - \frac{3}{2}\pi\right),\ \ y(0) = 0,\ \ y'(0) = 0$

3. Use the Laplace transform to solve the given system of differential equations.

   i) $\frac{dx}{dt} = -x + y\ ,\ \frac{dy}{dt} = 2x$

   $x(0) = 0,\ y(0) = 1$

   ii) $\frac{dx}{dt} + 3x + \frac{dy}{dt} = 1\ ,\ \frac{dx}{dt} - x + \frac{dy}{dt} - y = e^t$

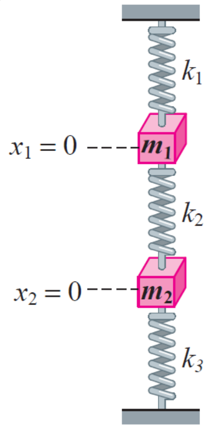
   $x(0) = 0,\ y(0) = 0$

   iii) $\frac{dx}{dt} - 4x + \frac{d^3y}{dt^3} = 6 \sin t\ ,\ \frac{dx}{dt} + 2x - 2\frac{d^3y}{dt^3} = 0$

   $x(0) = 0,\ y(0) = 0$

   $y'(0) = 0,\ y''(0) = 0$

4. Two masses m_1 and m_2 are connected to three springs of negligible mass having spring constants k_1, k_2 and k_3, respectively.

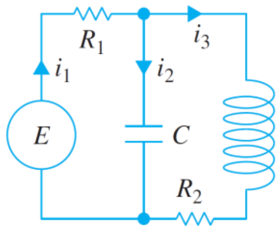


Let x_1 and x_2 represent displacements of masses m_1 and m_2 from their equilibrium positions. The motion of the coupled system is represented by the system of second-order differential equations:

$$m_1 \frac{d^2x_1}{dt^2} = -k_1x_1 + k_2(x_2 - x_1)$$
$$m_2 \frac{d^2x_2}{dt^2} = -k_2(x_2 - x_1) - k_3x_2$$

Using Laplace transform to solve the system when $k_1 = 1,\ k_2 = 1,\ k_3 = 1,\ m_1 = 1,\ m_2 = 1$ and $x_1(0) = 0,\ \ x_1'(0) = -1,\ x_2(0) = 0,\ x_2'(0) = 1.$

5. The system of differential equations for the charge on the capacitor $q(t)$ and the current $i_3(t)$ in the electrical network shown below is



$$R_1\frac{dq}{dt} + \frac{1}{C}q + R_1i_3 = E(t)$$
$$L\frac{di_3}{dt} + R_2i_3 - \frac{1}{C}q = 0$$

Find the charge on the capacitor $q(t)$ using Laplace transform when $L = 1H,\ R_1 = 1\Omega,\ R_2 = 1\Omega,\ C = 1F,\ E(t) = \begin{cases} 0, & 0 < t < 1 \\ 50e^{-t}, & t \geq 1 \end{cases},\ i_3(0) = 0,\ q(0) = 0.$